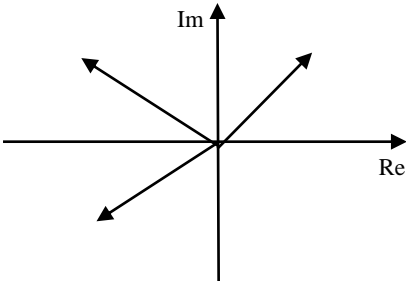




Mark Scheme (Results)

Summer 2022

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F1 (WFM01)
Paper 01

Question Number	Scheme	Notes	Marks
1(a)	$z_1 = 3 + 3i \quad z_2 = p + qi \quad p, q \in \mathbb{R}$		
	$ z_1 = \sqrt{3^2 + 3^2}$ $ z_1 z_2 = z_1 z_2 \Rightarrow z_2 \sqrt{18} = 15\sqrt{2} \Rightarrow z_2 = \dots$	Attempts $ z_1 $ using Pythagoras and uses $ z_1 z_2 = z_1 z_2 $ to find $ z_2 $	M1
	$ z_2 = 5$	Cao	A1
			(2)
ALT	$ z_1 z_2 = 15\sqrt{2}$ $ (3p - 3q) + i(3p + 3q) = 15\sqrt{2}$ $\sqrt{18p^2 + 18q^2} = 15\sqrt{2}$ $p^2 + q^2 = 25$ $ z_2 = \sqrt{p^2 + q^2} = 5$	Uses $ z_1 z_2 = z_1 z_2 $ to reach $p^2 + q^2 = \dots$	M1 A1 (2)
(b)	$ z_2 = 5 \Rightarrow p^2 + q^2 = 25$ $\Rightarrow (-4)^2 + q^2 = 25 \Rightarrow q = \dots$	Uses $p^2 + q^2 = "5"'^2$ with $p = \pm 4$ leading to a value for q .	M1
	$q = \pm 3$	Both values. Must be clear $p = 4$ has not been used	A1
			(2)
(c)	 <p>Points to be in the correct quadrants and either with correct numbers on the axes or labelled correctly</p>	3 + 3i plotted correctly and labelled Vectors/ lines not needed; point(s) alone are sufficient	B1
		A conjugate pair plotted correctly following through their q .	B1ft
			(2)
			Total 6

Question Number	Scheme	Notes	Marks
2	$f(x) = 10 - 2x - \frac{1}{2\sqrt{x}} - \frac{1}{x^3} \quad x > 0$		
(a)	$f(0.4) = -7.21\dots, f(0.5) = 0.292\dots$	Attempts both $f(0.4)$ and $f(0.5)$	M1
	Sign change (positive, negative) and $f(x)$ is continuous therefore (a root) α is between $x = 0.4$ and $x = 0.5$	Both $f(0.4) = \text{awrt } -7$ and $f(0.5) = \text{awrt } 0.3$, sign change and conclusion. Must mention continuity. Can have $f(0.4) \times f(0.5) < 0$ instead of “sign change”	A1
			(2)
(b)	$f'(x) = -2 + \frac{1}{4}x^{-\frac{3}{2}} + 3x^{-4}$	$x^n \rightarrow x^{n-1}$ in at least 1 term other than 10	M1
		2 of the 3 terms shown correct	A1
		All correct	A1
			(3)
(c)	$x_1 = 0.5 - \frac{f(0.5)}{f'(0.5)} = 0.5 - \frac{0.29289321\dots}{46.70710678\dots}$	Correct application of Newton-Raphson	M1
	$= 0.494$	Correct value 3dp. A correct derivative must have been used	A1
			(2)
(d)	$\frac{4.9 - \beta}{ f(4.9) } = \frac{\beta - 4.8}{f(4.8)} \Rightarrow \beta = \dots$	Uses a correct interpolation method (Signs to be correct)	M1
	$\beta = 4.883$	Correct value 3dp unless penalised in (c)	A1
			(2)
ALT 1	$\beta = \frac{a f(b) + b f(a) }{ f(a) + f(b) }$ $\beta = \frac{4.8 \times 0.0344 + 4.9 \times 0.1627}{0.0344 + 0.1627} = \dots$	Uses a correct interpolation method (Signs to be correct)	M1
	$\beta = 4.883$	Correct value 3dp unless penalised in (c)	A1
			(2)
ALT 2	Gradient $= \frac{-0.0344 - 0.1627}{4.9 - 4.8} = -1.971$ Equation of line: $y - 0.1627 = -1.971(x - 4.8)$ or $y = -1.971x + 9.6235$ Substitute $y = 0$ $x = \dots$	Complete method for line equation followed by substitution to obtain a value for x	M1
	$\beta = 4.883$	Correct value 3dp unless penalised in (c)	A1
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
3(a)	$\mathbf{M}^{-1} = \frac{1}{5k-3k} \begin{pmatrix} 5 & -k \\ -3 & k \end{pmatrix}$	Attempts $\mathbf{M}^{-1} = \frac{1}{\det \mathbf{M}} \times \text{adj}(\mathbf{M})$ Either part correct but $\text{adj}(\mathbf{M}) = \mathbf{M}$ scores M0	M1
	$= \frac{1}{2k} \begin{pmatrix} 5 & -k \\ -3 & k \end{pmatrix} \text{ or } \begin{pmatrix} \frac{5}{2k} & -\frac{1}{2} \\ \frac{-3}{2k} & \frac{1}{2} \end{pmatrix}$	Correct matrix $2k$ must be seen for this mark	A1
			(2)
(b)	$(\mathbf{MN})^{-1} = \mathbf{N}^{-1}\mathbf{M}^{-1} = \frac{1}{2k} \begin{pmatrix} k & k \\ 4 & -1 \end{pmatrix} \begin{pmatrix} 5 & -k \\ -3 & k \end{pmatrix}$	Applies $(\mathbf{MN})^{-1} = \mathbf{N}^{-1}\mathbf{M}^{-1}$	M1
	$= \frac{1}{2k} \begin{pmatrix} 2k & 0 \\ 23 & -5k \end{pmatrix} \text{ or e.g. } \begin{pmatrix} 1 & 0 \\ \frac{23}{2k} & \frac{-5}{2} \end{pmatrix}$	Correct matrix	A1
			(2)
ALT (b)	Find \mathbf{N} (ie inverse of \mathbf{N}^{-1}) Find $\mathbf{MN} = -\frac{1}{5k} \begin{pmatrix} -5k & 0 \\ -23 & 2k \end{pmatrix}$ Find $(\mathbf{MN})^{-1}$ $= \frac{1}{2k} \begin{pmatrix} 2k & 0 \\ 23 & -5k \end{pmatrix} \text{ or e.g. } \begin{pmatrix} 1 & 0 \\ \frac{23}{2k} & \frac{-5}{2} \end{pmatrix}$	Complete method needed Correct matrix	M1 A1
			(2)
			Total 4

Question Number	Scheme	Notes	Marks
4	$f(z) = 2z^4 - 19z^3 + Az^2 + Bz - 156$		
(a)	$(z =) 5 + i$	Correct complex number	B1
			(1)
	Mark (b) and (c) together – ignore any labelling seen. Award marks in the order given for their choice of method		
(b)/(c) With (b) first	$z = 5 \pm i \Rightarrow (z - (5 + i))(z - (5 - i)) = \dots$ Or e.g. Sum of roots = 10 Product of roots = 26	Correct strategy to find the quadratic factor using the conjugate pair	M1
	$z^2 - 10z + 26$	Correct quadratic	A1
	$f(z) = (z^2 - 10z + 26)(2z^2 + \dots z + k)$	Attempts to find the other quadratic. May use inspection (apply rules for quadratic factorisation ie " $26 \mid k = 156$ ") or e.g. long division with quotient $2z^2 + \dots z + \dots$	M1
	NB long division gives quotient $2z^2 + z + (A - 42)$ and remainder $(10A + B - 446)z + 936 - 26A$		
	$2z^2 + z - 6$	Correct quadratic	A1
	$\Rightarrow z = \frac{3}{2}, -2, (5 \pm i)$	Correct real roots. The complex roots do not have to be stated.	A1
			(5)
	$f(z) = (z^2 - 10z + 26)(2z^2 + z - 6) = \dots$	Multiplies out both quadratics or extracts the terms needed	M1
	$A = 36, B = 86$	Correct values (can be seen in the quartic equation)	A1
			(2)
			Total 8
(b)/(c) With (c) first	$952 + 960i - 2090 - 24A + 10Ai + 5Bi - 156 = 0$	Substitute $(5 + i)$ into the quartic (by calculator) and equate real and imag parts (can be done with $(5 - i)$)	M1
	$-1294 + 24A + 5B = 0$ $-446 + 10A + B = 0$	Correct equations	A1
	$A = 36 \quad B = 86$	M1 Solve simultaneously A1 One correct A1 Both correct	M1 A1A1
			(5)
	$2z^4 - 19z^3 + 36z^2 + 86z - 156 = 0$ $z = \dots$	Solve the equation by long division, inspection or by calculator	M1
	$\Rightarrow z = \frac{3}{2}, -2, (5 \pm i)$	Correct real roots. The complex roots do not have to be stated.	A1
			(2)
			Total 8

Question Number	Scheme	Notes	Marks
5	$2x^2 - 3x + 5 = 0$		
(a)	$\alpha + \beta = \frac{3}{2}, \quad \alpha\beta = \frac{5}{2}$	Both	B1
			(1)
(b)(i)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	Uses a correct identity	M1
	$= \left(\frac{3}{2}\right)^2 - 2\left(\frac{5}{2}\right) = -\frac{11}{4} (= -2.75)$	Correct value Allow to come from $\alpha + \beta = -\frac{3}{2}$	A1
(ii)	$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$	Reaches an identity ready for substitution	M1
	$= \left(\frac{3}{2}\right)^3 - 3\left(\frac{3}{2}\right)\left(\frac{5}{2}\right) = -\frac{63}{8} (= -7.875)$	Correct value	A1
			(4)
(c)	Sum = $\alpha^3 + \beta^3 - (\alpha + \beta) = -\frac{63}{8} - \frac{3}{2} \left(= -\frac{75}{8} \right)$	Attempts sum Allow eg $(\alpha^3 - \beta) + (\beta^3 - \alpha)$ followed by $(\alpha^3 + \beta^3) + (\alpha + \beta) = \dots$	M1
	Prod = $(\alpha\beta)^3 - \alpha^4 - \beta^4 + \alpha\beta$ and $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$	Expands $(\alpha^3 - \beta)(\beta^3 - \alpha)$ and uses a correct identity for $\alpha^4 + \beta^4$	M1
	Alt identities: $\alpha^4 + \beta^4 =$ $(\alpha + \beta)^4 - 4\alpha\beta(\alpha^2 + \beta^2) - 6\alpha^2\beta^2; \alpha^4 + \beta^4 = (\alpha^3 + \beta^3)(\alpha + \beta) - \alpha\beta(\alpha^2 + \beta^2)$		
	$(\alpha\beta)^3 - \alpha^4 - \beta^4 + \alpha\beta = \left(\frac{5}{2}\right)^3 + \frac{5}{2} - \left(\left(-\frac{11}{4}\right)^2 - 2\left(\frac{5}{2}\right)^2\right) = \frac{369}{16}$		A1
	$x^2 + \frac{75}{8}x + \frac{369}{16} (= 0)$	Applies $x^2 - (\text{their sum})x + \text{their prod} (= 0)$	M1
	$16x^2 + 150x + 369 = 0$	Allow any integer multiple	A1
			(5)
			Total 10

Question Number	Scheme	Notes	Marks
6(a)	$x = 9t^2, y = 18t \Rightarrow \frac{dy}{dx} = \frac{18}{18t}$ <p style="text-align: center;">or</p> $y^2 = 36x \Rightarrow 2y \frac{dy}{dx} = 36 \Rightarrow \frac{dy}{dx} = \frac{18}{y} = \frac{18}{18t}$ <p style="text-align: center;">or</p> $y^2 = 36x \Rightarrow y = 6\sqrt{x} \Rightarrow \frac{dy}{dx} = \frac{3}{\sqrt{x}} = \frac{3}{3t}$	<p>Correct $\frac{dy}{dx}$ in terms of t</p> <p>There must be evidence of use of calculus ($\frac{dy}{dx} = \frac{1}{t}$ with no working scores B0)</p>	B1
	$m_T = \frac{1}{t} \Rightarrow m_N = -t$	Correct use of the perpendicular gradient rule.	M1
	$y - 18t = -t(x - 9t^2)$	Correct straight line method for the normal. Must use their perpendicular gradient – not dy/dx. (Any complete method – use of $y = mx + c$ requires an attempt at “c”)	dM1
	$y + tx = 9t^3 + 18t^*$	Cso All previous marks must have been earned	A1*
			(4)
(b)	$x = 54, y = 0 \Rightarrow 54t = 9t^3 + 18t$ $\Rightarrow 9t^3 - 36t = 0$	Substitutes $x = 54$ and $y = 0$ into the equation from part (a) and attempts to collect terms.	M1
	$9t^3 - 36t = 0 \Rightarrow 9t(t^2 - 4) = 0$ $\Rightarrow t = \pm 2 \Rightarrow y \pm 2x = 9(\pm 2)^3 + 18(\pm 2)$	Solves to obtain at least one non zero value for t and attempts at least one normal equation	dM1
	$y = -2x + 108$ or $y = 2x - 108$	One correct equation in any equivalent form	A1
	$y = -2x + 108$ and $y = 2x - 108$	Both correct and in the required form	A1
			(4)
(c)	$x = -9 \Rightarrow y = 18 + 108$ or $-18 - 108$	Uses $x = -9$ to find the y coordinate of A or B	M1
	Area = $\frac{1}{2} \times 252 \times 18$	Fully correct strategy for the area Award M0 if their x coord of the focus is not doubled	M1
	= 2268	Cao	A1
			(3)
			Total 11
ALT	<p>Last 2 marks by “shoelace” method:</p> <p>eg $\begin{vmatrix} 1 & -9 & 9 & -9 \\ 2 & 126 & 0 & -126 & 126 \end{vmatrix}$</p> <p>$= \frac{1}{2}(9 \times -126 - 9 \times 126 - (-9 \times -126 + 9 \times 126))$</p> <p>= 2268</p>	<p>Their coordinates with first and last the same</p> <p>$\frac{1}{2}$ must be included</p> <p>Attempt to expand also needed</p> <p>Must be positive</p>	<p>M1</p> <p>A1</p>

Question Number	Scheme	Notes	Marks
7(a)	$\mathbf{A}^2 = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$	Correct matrix	B1
			(1)
(b)	Rotation -60° (anticlockwise) about the origin	Rotation	M1
		-60° (anticlockwise) (Or 60° clockwise or 300° (anticlockwise)) about (0, 0)	A1
			(2)
(c)	$n = 12$	Cao but can be embedded ie $A^{12} = I$	B1
			(1)
(d)	$\mathbf{B} = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$	Correct matrix	B1
			(1)
(e)	$\mathbf{C} = \mathbf{BA} = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$	Multiplies the right way round.	M1
	$\mathbf{C} = \begin{pmatrix} -2\sqrt{3} & -2 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$	Correct matrix Accept unsimplified	A1
			(2)
(f)	$\det \mathbf{C} = -2\sqrt{3} \times -\frac{\sqrt{3}}{2} - \frac{1}{2}(-2) = 4$ So area of P is $\frac{20}{\det \mathbf{C}} = \dots$	Attempts determinant of \mathbf{C} (or deduces area scale factor is 4) and divides into 20	M1
	$= 5$	Cao Must follow a correct matrix in (e)	A1
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
8	$\sum_{r=0}^n (r+1)(r+2)$		
(a)	$\sum_{r=0}^n r^2 + 3r + 2 = 2 + \frac{1}{6}n(n+1)(2n+1) + \frac{3}{2}n(n+1) + 2n$ <p>M1: Attempt to use at least one of the standard formulae correctly</p> <p>A1: For $\frac{1}{6}n(n+1)(2n+1) + \frac{3}{2}n(n+1) + (2n \text{ or } 2n+2)$</p> <p>A1: Fully correct expression</p>		M1A1A1
	$\frac{1}{6}n(n+1)(2n+1) + \frac{3}{2}n(n+1) + 2n + 2 = (n+1) \left[\frac{1}{6}n(2n+1) + \frac{3}{2}n + 2 \right]$ <p>Attempt to factorise $(n+1)$</p> <p>It is a “show” question so this must be seen (in any equivalent form).</p> <p>If their expression does not allow for factorising $(n+1)$ score M0</p>		M1
	$\frac{1}{3}(n+1)[n^2 + 5n + 6]$	May obtain a cubic and extract a different factor ie $n+2$ or $n+3$	
	$\frac{1}{3}(n+1)(n+2)(n+3)^*$	Cso At least one intermediate step in the working must be seen.	A1*
			(5)
(a) Way 2	$\sum_{r=0}^n (r+1)(r+2) = \sum_{r=1}^{n+1} r(r+1)$ $= \sum_{r=1}^{n+1} r^2 + r = \frac{1}{6}(n+1)(n+2)(2(n+1)+1) + \frac{1}{2}(n+1)(n+2)$ <p>M1: Attempt to use at least one of the standard formulae correctly with $n = n+1$</p> <p>A1: For $\frac{1}{6}(n+1)(n+2)(2(n+1)+1)$ or $\frac{1}{2}(n+1)(n+2)$</p> <p>A1: Fully correct expression</p>		M1A1A1
	$\frac{1}{6}(n+1)(n+2)(2(n+1)+1) + \frac{1}{2}(n+1)(n+2) = (n+1) \left[\frac{1}{6}(n+1)(2n+3) + \frac{1}{2}(n+2) \right]$ <p>Attempt to factorise $(n+1)$ (see additional comments above)</p>		M1
	$\frac{1}{3}(n+1)[n^2 + 5n + 6]$	May obtain a cubic and extract a different factor ie $n+2$ or $n+3$	
	$\frac{1}{3}(n+1)(n+2)(n+3)^*$	Cso At least one intermediate step in the working must be seen.	A1*
(b)	Upper limit = 99	Correct upper limit	B1
	$10 \times 11 + 11 \times 12 + 12 \times 13 + \dots + 100 \times 101 = \sum_{r=0}^{99} (r+1)(r+2) - \sum_{r=0}^8 (r+1)(r+2)$ <p>Fully correct strategy for the sum using their upper limit for the first sum and upper limit 8 for the second in the result from (a). Lower limits 0 or 1</p>		M1
	$= \frac{1}{3}(100)(101)(102) - \frac{1}{3}(9)(10)(11)$ <p>= 343 070</p>	Correct value	A1

			(3)
			Total 8

Question Number	Scheme	Notes	Marks
9(i)	$u_n = 5 \times 2^{n-1} - n \times 2^n$		
	$n = 1 \Rightarrow u_1 = 5 \times 2^0 - 1 \times 2 = 3$ (Shows the result is true for $n = 1$)		B1
	Assume true for $n = k$ so that $u_k = 5 \times 2^{k-1} - k \times 2^k$		
	$u_{k+1} = 2(5 \times 2^{k-1} - k \times 2^k) - 2^{k+1}$	Attempts u_{k+1} using the recurrence relationship	M1
	$= 5 \times 2^k - k \times 2^{k+1} - 2^{k+1}$	Correct expanded expression	A1
	$= 5 \times 2^k - (k+1)2^{k+1}$	Achieves this result with no errors	A1
	If the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n .		A1cso
	The final mark depends on all except the B mark, though a check for $n = 1$ must have been attempted.		
			(5)
(ii)	$f(n) = 5^{n+2} - 4n - 9$		
	$f(1) = 125 - 4 - 9 = 112 = 16 \times 7$	Shows $f(1)$ is divisible by 16 Either of 112 or 16×7 must be seen	B1
	Assume true for $n = k$ so that $5^{k+2} - 4k - 9$ is divisible by 16		
	$f(k+1) = 5^{k+3} - 4(k+1) - 9$	Attempts $f(k+1)$	M1
	$= 5 \times (5^{k+2} - 4k - 9) + \dots$	Attempts to express in terms of $f(k)$	dM1
	$= 5 \times (5^{k+2} - 4k - 9) + 16k + 32$	Correct expression for $f(k+1)$	A1
	If the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n .		A1cso
	The final mark depends on all except the B mark, though a check for $n = 1$ must have been attempted		(5)
			Total 10

ii ALT 1	$f(1) = 125 - 4 - 9 = 112 = 16 \times 7$	Shows $f(1)$ is divisible by 16 Either of 112 or 16×7 must be seen	B1
	Assume $5^{k+2} - 4k - 9$ is divisible by 16		
	$f(k+1) - mf(k) = 5^{k+3} - 4(k+1) - 9 - m(5^{k+2} - 4k - 9)$ Attempt $f(k+1) - mf(k)$		M1
	$= (5-m)(5^{k+2} - 4k - 9) + \dots$	Attempts to express in terms of $f(k)$	dM1
	$f(k+1) = 5 \times (5^{k+2} - 4k - 9) + 16k + 32$	Correct expression for $f(k+1)$	A1
	If the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n .		A1cso
	The final mark depends on all except the B mark, though a check for $n = 1$ must have been attempted		
ii ALT 2	$f(1) = 125 - 4 - 9 = 112 = 16 \times 7$	Shows $f(1)$ is divisible by 16 Either of 112 or 16×7 must be seen	B1
	Assume $5^{k+2} - 4k - 9$ is divisible by 16		
	$f(k+1) - f(k) = 5^{k+3} - 4(k+1) - 9 - (5^{k+2} - 4k - 9)$ Attempt $f(k+1) - f(k)$		M1
	$f(k+1) - f(k) = 5 \times 5^{k+2} - 5^{k+2} - 4k - 4 - 9 + 4k + 9$ $= 4 \times 5^{k+2} - 4 = 4(5^{k+2} - 1)$ Obtains a simplified expression for the difference and attempts to prove $(5^{k+2} - 1)$ is divisible by 4 using induction		dM1
	Correct proof for $(5^{k+2} - 1)$ being divisible by 4 and states that thus as the difference is divisible by 16, $f(k+1)$ is divisible by 16		A1
	If the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n .		A1 cso
	The final mark depends on all except the B mark, though a check for $n = 1$ must have been attempted		
ii ALT 3	$f(1) = 125 - 4 - 9 = 112 = 16 \times 7$	Shows $f(1)$ is divisible by 16 Either of 112 or 16×7 must be seen	B1
	$f(k)$ is divisible by 16 so set $f(k) = 16\lambda$		
	$5^{k+2} = 16\lambda + 4k + 9$		M1
	$f(k+1) = 5^{k+3} - 4(k+1) - 9$ $= 5 \times 5^{k+2} - 4k - 13 = 5(16\lambda + 4k + 9) - 4k - 13$	Expresses $f(k+1)$ in terms of λ and k and collects terms	dM1
	$= 80\lambda + 16k + 32$	Correct expression May have factor of 16 taken out	A1
	If the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n .		A1cso
	The final mark depends on all except the B mark, though a check for $n = 1$ must have been attempted		